

Quantum Key Distribution with Screening and Analyzing

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We propose a quantum key distribution scheme by using screening angles and analyzing detectors which enable to notice the presence of Eve who eavesdrops the quantum channel, as the revised protocol of the recent quantum key distribution [Phys. Rev. Lett. 95, 040501 (2005)]. We discuss the security of the proposed quantum key distribution against various attacks including impersonation attack and Trojan Horse attack.

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Quantum key distribution (QKD) is to find a way which enable to share the secret information between distant parties by transmitting quantum states. The security of QKD is guaranteed by the law of quantum physics, while that of the classical key distribution or cryptography is done by the computational cost of the underling mathematical problems.

Bennet-Brassard (BB84)[1] proposed the first QKD protocol which uses 4 quantum states and Ekert (E91) [2] suggested basing the security of the two-qubit protocol on Bell's inequality [3]. On the other hand, Boström and Felbinger' [4] initiates a new type of QKD which generates the key in deterministic way by passing the qubit in two-way. Lucamarini and Mancini's protocol [5] is also classified in this type QKD. The deterministic feature of the two-way protocols enables to apply the direct encoding but in some case it was considered as a vulnerable point.

The situation in regard to the trend of QKD looks contradictory in the sense that QKD should be implemented with single photon or sufficiently weak pulse to guarantee the security, even though the coding should be done with strong pulse to send the signal to longer distance. In this regard, Kye et al. [6] has proposed 3-way type QKD protocol which enables to send not-so-weak coherent pulses by loading the encoding onto the qubit with perfect random polarization. Since the first version of the protocol is vulnerable against the impersonation attack when it is implemented with single pulse, they have proposed the protocol using two coherent pulses. Unfortunately, there has been several attacks [7, 8, 9] and the revised protocols [9, 10, 11] are followed. Even though the revised version of the protocol is secure in present, the fact that two coherent pulses are used to encode qubit has been considered as the attack point [7, 8, 10, 11].

In this paper, keeping the basic philosophy of QKD in Ref. [6], we propose a new scheme which blocks up all known attacks [7, 8] and the composite attack which is composed of PNS and Trojan Horse attack. The proposed QKD protocol is described as follows:

(P.1) Alice announces a set $S(N)$ which has N number

of screening angles,

$$S(N) = \{\alpha_1, \dots, \alpha_N\}, \quad (1)$$

where the screening angle α_i is defined as $\alpha_i = i\pi/2(N+1)$. Alice prepares the analyzing detectors (AD) in her setup (see Fig. 1).

(P.2) Alice prepares a qubit with arbitrary angle θ .

$$|\theta\rangle, \quad (2)$$

She sends the qubit to Bob.

(P.3) Bob choose a random angle ϕ and a screening angle α_b . He acts $U(\phi + \alpha_b)$ on the received qubit. The qubit becomes

$$|\theta + \phi + \alpha_b\rangle, \quad (3)$$

where Bob occasionally takes ϕ as an analyzing angle $\phi^* \in \{0, \pi/2\}$ with the probability p_a . Bob sends the qubit to Alice.

(P.4) After receiving the qubit, Alice applies $U(-\theta + (-1)^k\pi/4 + \alpha_a)$, where α_a is a chosen screening angle. The qubit becomes

$$|\phi + (-1)^k\pi/4 + \alpha_a + \alpha_b\rangle, \quad (4)$$

The fraction $(1-t)$ of the photons in the qubit clicks the Alice's AD D_1 and D_2 . If Bob takes $\phi = \phi^*$ and screening angles α_a and α_b are matched as

$$\alpha_a + \alpha_b = \pi/2, \quad (5)$$

then the qubit incoming to Alice's detector is $|\phi^* + (-1)^k\pi/4 + \pi/2\rangle$. Thus the outcome on the Alice's detector O_a and Alice's encoded key k are related by

$$O_a = k \oplus (2\phi^*/\pi) \oplus 1. \quad (6)$$

The above equation shows Alice's integrity condition in AD. The remaining fraction t of the photons in qubit is sent to Bob.

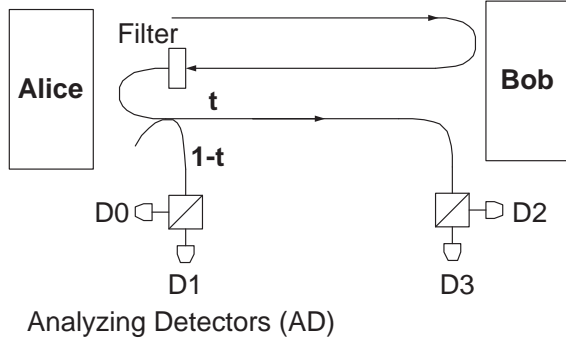


FIG. 1: Setup of the proposed QKD.

(P.5) Bob applies $U(-\phi)$ on the received qubit. The qubit becomes $|(-1)^k\pi/4 + \alpha_a + \alpha_b\rangle$. Bob measures the received qubit in $(+\pi/4, -\pi/4)$ basis and get the outcome O_b . The Bob's outcome is definitely correlated with the encoded key k only when the screening angles are matched as Eq. (5) and the key k and Bob's outcome O_b are related by

$$O_b = k \oplus 1. \quad (7)$$

(P.6) After repeating M times from (P.2) to (P.5), Alice and Bob disclose the series of screening angles α_a and α_b and the sequence of analyzing angle with ϕ^* and the value of ϕ^* . Alice and Bob create the key k_a and k_b only with the sequence satisfying Eq. (5) and exchange the hash values $h(k_a)$ and $h(k_b)$ of the created k_a and k_b [6]. If $h(k_a) = h(k_b)$ and Eq. (6) is verified for all analyzing angles ϕ^* , key creation is finished else Alice and Bob start again from (P.1).

In general, the probability of the event which is satisfied with Eq. (6) is $N/N^2 = 1/N$ thus the key creation rate is proportional to $1/N$ in this QKD. For example, the simplest implementation is plausible with $N = 2$. In that case, $S(2) = \{\pi/6, 2\pi/6\}$ and 50% of the transmitted qubits are satisfied with the matching condition Eq. (5). So Alice and Bob can utilize 50% of transmitted qubits in creation of key. The screening angles and AD play an important role to detect the presence of Eve. As we shall see, the screening angle unable Eve to relay the qubit without inducing the measurement error in Bob and AD blocks up general Trojan Horse attack and more sophisticated one combined with PNS.

Since Alice and Bob only utilize the qubit event satisfying Eq. (6) in creation of the key, this QKD is not deterministic protocol. That is neither Alice nor Bob expect the created key and it is determined only after the announcement of screening angles, which is the original concept of BB84 [1]. This is different feature compared with the original QKD in Ref. [6]. Here we shall show that the proposed QKD with screening angle

and AD is secure against the known attacks [7, 8]

Security against Impersonation attack:

In regard to the impersonation attack, we shall consider the case that Alice turns off AD by setting the transmission coefficient $t = 1$ for the demonstration of the role of screening angles clearly.

(1) Single photon Implementation:

- (A.1) After (P.2), Eve intercepts and stores the pulse from Alice in set E_1 . Thus Eve has $E_1 = \{|\theta\rangle\}$. Eve sends to Bob a pulse originally prepared by her with random angles θ' .
- (A.2) After step (P.3), Eve intercepts both pulses from Bob and stores them in set E_2 after compensating with the angles $-\theta'$. Eve then has $E_2 = \{|\phi + \alpha_b\rangle\}$. Eve send E_1 qubit to Alice.
- (A.3) After step (P.4), Eve intercepts the returning qubit $|(-1)^k\pi/4 + \alpha_a\rangle$. Eve need to guess α_a to measure the qubit without error. If Eve's guessing is correct, Eve can read Alice's encoding and Eve get the outcome of the measurement $O_e = k$. Eve encode the O_e into E_2 qubit then the qubit becomes $E'_2 = \{|\phi + (-1)^k\pi/4 + \alpha_b\rangle\}$. Eve send the qubit to Bob.
- (A.4) After the step (P.5), the receive qubit becomes $E''_2 = \{|(-1)^k\pi/4 + \alpha_b\rangle\}$. Thus the Bob measurement includes the error induced by Eve's impersonation as follows:

$$\text{IE}(N) \sim \sum_{i=1}^N \sin^2(\alpha_i - \pi/2),$$

which is the lowest bound of the error due to Eve's impersonation. Thus it should be noticed in the key verification stage of (P.6). If Eve's guessing was not correct, Eve reads incorrect encoding and the error is additionally propagated into the E'_2 and Bob's key. It also should be noticed in the stage of (P.6).

(2) Pulse implementation:

In this case, Eve's attack strategy is different from that of the single photon implementation. Eve can use beam splitter to separate the pulse into two parts and Eve tries to analyze the pulse measuring it on both bases $(\alpha_1 + \pi/4, \alpha_1 - \pi/4)$ and $(\alpha_2 + \pi/4, \alpha_2 - \pi/4)$. However in this experiment Eve can not get the conclusive results because two bases are not orthogonal with each other. Thus it eventually induces the error in Bob's key as a results of Eve's relay based on inconclusive result.

If Alice turns on the AD with $t < 1$, Eve's impersonation without knowing the ϕ should violate the

Alice's integrity condition Eq. (6). So Alice should notice the presence of Eve more easily.

Security against photon number splitting (PNS) attack:

Since Alice and Bob announce the screening angles in (P.6), the security against PNS attack determined by the two random angle θ and ϕ . Thus the security is almost same with that of single pulse implementation of QKD in Ref. [6]. The only difference is that the random angle ϕ is partially known to Eve when Bob chooses the analyzing angle $\phi = \phi^*$.

Security against Trojan Horse type attack:

To eavesdrop the key, Eve needs to distinguish the her injected photon from those in the pulse and separate it out correctly. Thus Eve necessarily marks the injected photon by using time delay or frequency shift [12]. However it is known that this type of marking can be blocked with properly designed apparatuses and filters [12]. Accordingly, we assume that Alice has a properly designed filter as in Fig. 1.

It is clear that the proposed QKD is robust against simple Trojan Horse attack, because in (P.4) Alice's encoding is performed with the random angle compensation of θ such that $U(-\theta + (-1)^k \pi/4 + \alpha_a)$. Accordingly, the information gain for Eve, who attacks quantum channel with independent Trojan Horse state $|\eta\rangle$, is zero because the separated qubit always has the random polarization. Thus the general Trojan Horse attacks discussed in Ref. [12] are not useful for Eve in our QKD.

However Eve can consider the following strategy which combines PNS and Trojan Horse attack.

(B.1) After (P.2) Eve performs quantum measurement on the pulse and if the number of photons are larger than one, Eve separate out a photon and keeps it in $E_1 = \{|\theta\rangle\}$.

(B.2) Eve attaches the photon $|\theta\rangle$ onto the incoming pulse after (P.3). The state of incoming photon pulse is given by $|\theta + \phi + \alpha_b\rangle \otimes |\theta\rangle$. After (P.4) the outgoing pulse is given by

$$|(-1)^k \pi/4 + \phi + \alpha_a + \alpha_b\rangle \otimes |(-1)^k \pi/4 + \alpha_a\rangle. \quad (8)$$

(B.3) Eve keeps $E_2 = \{|(-1)^k \pi/4 + \alpha_a\rangle\}$ after separating the photon from the pulse. Eve reads the key value k by measuring E_2 after the announce of α_a in (P.6).

With this strategy, if Eve separates out the $|(-1)^k \pi/4 + \alpha_a\rangle$, she can eavesdrop the k after the announcing of α_a in (P.6) by measuring the separated qubit [15].

We can see that this strategy of Eve is easily noticed by the help of AD. The fraction $(1-t)$ of photons in the qubit in Eq. (8) clicks on Alice's AD. The mixed state

$|(-1)^k \pi/4 + \phi + \alpha_a + \alpha_b\rangle \otimes |(-1)^k \pi/4 + \alpha_a\rangle$ eventually give rise to the violation of integrity condition in Eq. (6) [14], because the equation holds only when the pure state $|(-1)^k \pi/4 + \phi + \alpha_a + \alpha_b\rangle$ is entered.

Eve may consider the attack proposed in Ref. [13] which attaches the standard state $|0\rangle$ instead of PNS photon $|\theta\rangle$ in (B.2) and calculates the polarization directly estimating maximal mean fidelity. The corresponding qubit is

$$|(-1)^k \pi/4 + \phi + \alpha_a + \alpha_b\rangle \otimes |-\theta + (-1)^k \pi/4 + \alpha_a\rangle. \quad (9)$$

This qubit also violate the Alice's integrity condition in AD (Eq. (6)) [14].

By announcing the larger number of screening angles N , Alice and Bob may get rid of possibility of the leakage of the information encoded on the qubit. To attack the protocol which uses the larger number of screening angle N , Eve need to prepare N separated pulse by using the beam splitter and to measure the pulses with N different nonorthogonal basis. Accordingly, the probability of which Eve get the conclusive result for relay is suppressed depending on N thus we can say that the security level of this scheme is proportional to N , while the key creation rate is proportional to $1/N$. We note the Bob's choice of the analyzing angle ϕ^* does not affect the key creation rate because ϕ^* is also one of the random angle that Bob can choose in (P.3). Alice and Bob establish the desired security level by controlling the number of screening angles N , appropriately. It can be summarized as follows:

$$\text{Key Creation Rate} \sim 1/N, \quad (10)$$

$$\text{Security Level} \sim N. \quad (11)$$

In conclusion, we have proposed the revised QKD protocol which enables to implement QKD with not-so-weak single pulse. We have shown that the screening angles and AD enable Alice and Bob not only to notice the impersonation attack but also to block up the Trojan Horse attack and the composite attack with PNS. The proposed QKD is simple to implement and provides tunable security level depending N .

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- [14] In AD, the events which click $D1$ and $D2$ simultaneously are also considered as the violation of integrity condition, because the value of O_a in Eq. (6) is inconclusive.
- [15] In this stage, we assume that Eve has identified injected photon as well as separated it successfully among the identical photons inside of coherent pulse by her unlimited technology, even though this is not the technical problem but principle one.